Dilations With Matrices

About the Lesson

In this activity, students use matrices to perform dilations centered at the origin of triangles. As a result, students will:

- Explore the effect of the scale factor on the size relationship between the pre-image and image of a polygon.

Vocabulary

- dilation
- pre-image
- scale factor
- matrix multiplication

Teacher Preparation and Notes

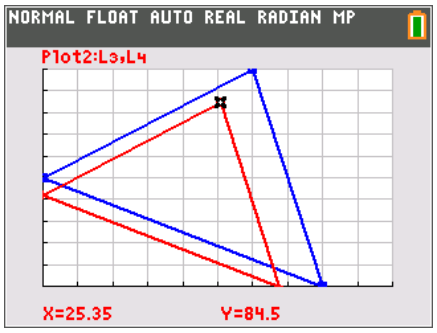
- It is assumed that student have a fundamental knowledge of matrices prior to this activity.
- This activity uses the concept of matrix multiplication when applying the scale factor on the sized relationship between the pre-image and image of a triangle.

Activity Materials

- Compatible TI Technologies:

TI-84 Plus*
TI-84 Plus Silver Edition*
TI-84 Plus C Silver Edition
TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

- Dilations_With_Matrices_Student.pdf
- Dilations_With_Matrices_Student.doc

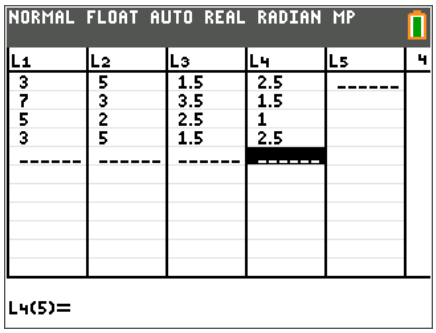
Problem 1 – Dilation Example

To begin this activity, students will draw two triangles with coordinates (3, 5), (7, 3), (5, 2) and (1.5, 2.5), (3.5, 1.5), (2.5, 1) using scatter plots.

Students will enter the following data into lists 1 through 4 by pressing [stat] [enter].

L1: 3, 7, 5, 3 **L3**: 1.5, 3.5, 2.5, 1.5

L2: 5, 3, 2, 5 **L4**: 2.5, 1.5, 1, 2.5



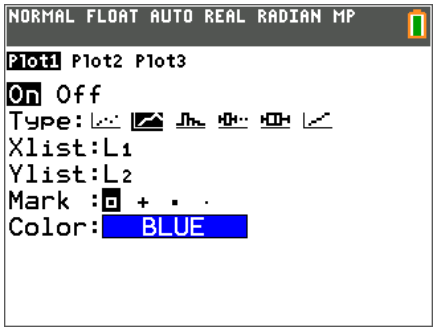
It may be helpful to remind students that **L1** contains the *x*-coordinates and **L2** contains the *y*-coordinates for the first triangle with the first point repeated. The same pattern holds for **L3** and **L4**.

Two connected scatter plots will be set up by assigning

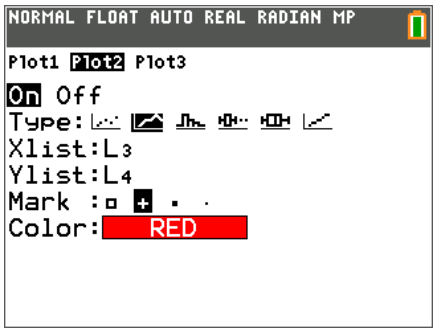
Plot 1 for **L1** and **L2** and Plot 2 for **L3** and **L4**.

To set up the first connected scatter plot, press [2nd] [stat plot] and select **Plot1**.

The settings for **Plot1** appear to the right.

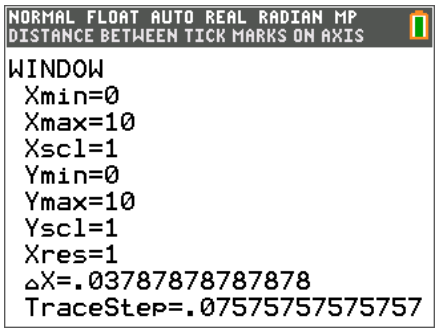


When drawing **Plot2**, set the marking as a plus sign as it will aid in distinguishing the two plots from one another.

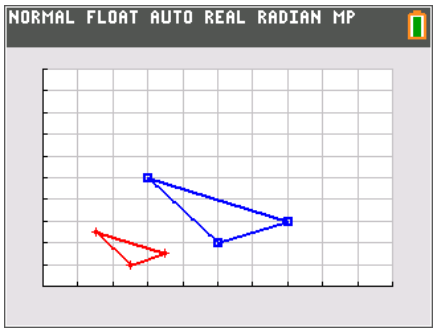


Set the viewing window by pressing [window] and match the settings for **Xmin**, **Xmax**, **Xscl**, **Ymin**, **Ymax**, and **Yscl** with those on screen to the right. Once that is done, press [graph] to view the two triangles.

Note: The values for **ΔX** and **TraceStep** are not important for this activity.



The triangle with the square vertex markings is the pre-image and the triangle with the plus vertex markings is the image under a dilation with scale factor of 0.5. Use [trace] to examine the coordinates of these two triangles.



Tech Tip: If your students are using the TI-84 Plus CE have them turn on the GridLine by pressing [2nd] [zoom][format] to change the graph settings. If your students are using TI-84 Plus, they could use GridDot.

1. What do you notice about the coordinates of the pre-image and image?

Answer: Each coordinate of the image is one-half that of the pre-image.

A dilation matrix is created by putting the scale factor on the diagonal of the matrix and leaving all other entries as zero.

The general dilation matrix with scale factor k is: $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$.

2. Using this information, write a matrix multiplication problem to determine the coordinates of the image of the triangle.

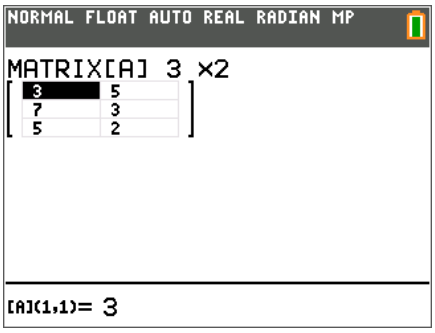
Answer: $\begin{bmatrix} 3 & 5 \\ 7 & 3 \\ 5 & 2 \end{bmatrix} \cdot \begin{bmatrix} .5 & 0 \\ 0 & .5 \end{bmatrix}$

Problem 2 – Scaling Up or Down

Students will now explore the effect the scale factor has on the triangle whose coordinates are in **Plot1**.

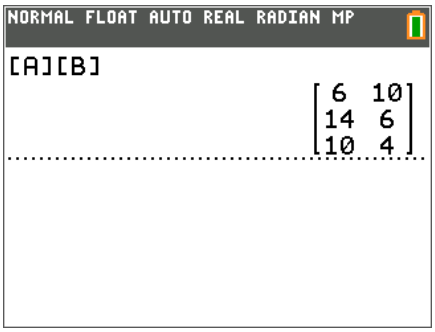
The three coordinates of the pre-image triangle can be stored as a matrix called *A* by pressing 2nd [matrix], arrowing over to the **Edit** menu, selecting **1:[A]**, and then entering the dimensions and entries of the matrix.

The screen to the right displays matrix *A* after the coordinates of the vertices have been entered.



Students will be asked to observe the effect a *k* value of 2 has on the triangle, by entering the dilation matrix $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ as matrix *B* and then multiplying [*A*][*B*] and then entering these new coordinates for the image into **L3** and **L4**.

Next, they are asked to try this with different values of *k* and see if there is a pattern.



3. Write a conjecture for how the scale factor, *k*, determines the size of the image.

Answer: The value of *k* acts as a scalar multiple to each coordinate value of the pre-image. For a value of $k > 1$, the image will be larger than the pre-image. For a value of $0 < k < 1$, the image will be smaller than the pre-image.

4. Using your conjecture, write a matrix multiplication problem for a triangle with coordinates (–7, –5), (–5, 4) and (2, –6) where the image is larger. Determine the coordinates of the vertices of your image triangle. (You may need to change the viewing window to observe the new image.)

Sample Answer: $\begin{bmatrix} -7 & -5 \\ -5 & 4 \\ 2 & -6 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -14 & -10 \\ -10 & 8 \\ 4 & -12 \end{bmatrix}$

5. Using your conjecture, write a matrix multiplication problem for a triangle with coordinates (–7, –5), (–5, 4) and (2, –6) where the image is smaller. Determine the coordinates of the vertices of your image triangle.

Sample Answer: $\begin{bmatrix} -7 & -5 \\ -5 & 4 \\ 2 & -6 \end{bmatrix} \cdot \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} = \begin{bmatrix} -3.5 & -2.5 \\ -2.5 & 2 \\ 1 & -3 \end{bmatrix}$

6. Using your conjecture, write a matrix multiplication problem a triangle with coordinates (–7, –5), (–5, 4) and (2, –6) where the image is equal. Determine the coordinates of the vertices of your image triangle.

Answer: $\begin{bmatrix} -7 & -5 \\ -5 & 4 \\ 2 & -6 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & -5 \\ -5 & 4 \\ 2 & -6 \end{bmatrix}$

Problem 3 – Fencing a Garden

Students are asked to find coordinates of the fence posts of a new garden. Students should be reminded that the scale factor is not equal to the ratio of the garden areas, but it is equal to the square root of this ratio. The reason for this is due to the fact that we need to know the scale for each axis in order to dilate the figure. Thus, the square root of the ratio of the two areas is $\sqrt{\frac{1250}{1750}} \approx 0.845$.

7. A gardener has fenced in a triangular garden with fence posts at (30, 100), (40, 0) and (0, 50). The area of the garden is 1750 square feet. After a year, the gardener has decided that his garden is too big to maintain. He now wants the size of the garden to be 1250 square feet. Help the gardener determine where his three fence posts should now be to create the garden using dilations and matrix multiplication.

Answer: $\begin{bmatrix} 30 & 100 \\ 40 & 0 \\ 0 & 50 \end{bmatrix} \cdot \begin{bmatrix} 0.845 & 0 \\ 0 & 0.845 \end{bmatrix} = \begin{bmatrix} 25.35 & 84.5 \\ 33.8 & 0 \\ 0 & 42.25 \end{bmatrix}$

The values in the image matrix may vary slightly due to rounding.