



Math Objectives

- Students will recognize that the sum of two central angles that combine to make a circle is 360° .
- Students will recognize that the ratio of the central angle to 360° determines the length of an arc.
- Students will recognize that the ratio of the central angle to 360° determines the area of a sector.
- Students will identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
- Students will derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
- Students will look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

- circumference
- area
- central angle (or sector angle)
- arc
- arc length
- arc measure
- sector

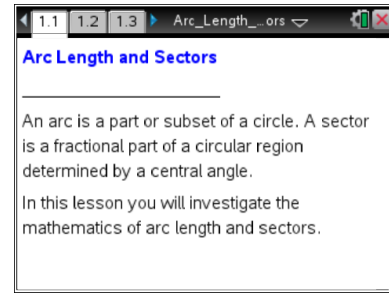
About the Lesson

- This lesson involves dragging a point around a circle to see how arcs and sectors change as the central angle defining them changes. As a result, students will:
 - Watch the central angles, sectors, and accompanying ratios change as they drag a point around a circle.
 - Specify the percentages (ratios) of circumference and area for various central angles.
 - Identify the connection between the central angle and the percentage of circumference and area used to determine arc length and sector area.



TI-Nspire™ Navigator™

- Use Class Capture to monitor progress.
- Use Quick Poll to assess progress.



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at <http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials>

Lesson Files:

Student Activity

- Arc_Length_and_Sectors_Student.pdf
- Arc_Length_and_Sectors_Student.doc

TI-Nspire document

- Arc_Length_and_Sectors.tns



Prerequisite Knowledge

- Students should be able to calculate circumference and area of a circle.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

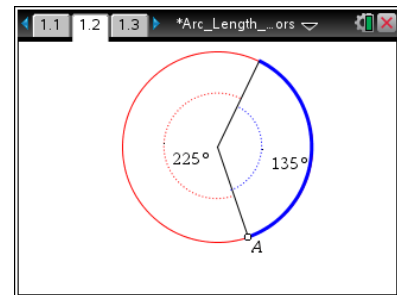
Discussion Points and Possible Answers



Tech Tip: If students experience difficulty dragging a point, check to make sure they have moved the arrow until it becomes a hand () getting ready to grab the point and close the hand (). When finished moving the point, press `esc` to release the point.

Move to page 1.2.

1. Drag point A around the given circle. What numerical fact do you observe about the measures of the two angles surrounding the center of the circle?



Answer: The sum of 2 angle measures is always 360° .

Teacher Tip: Have students rotate point A both clockwise and counterclockwise. If they don't immediately identify the sum, suggest that they make one of the angles an easy number, such as 10° . Or have them make it 0° , then increase in increments of 1° and observe the changes. Because of rounding, it is possible to not get exactly 360° . This can provide for good discussion.



TI-Nspire Navigator Opportunity: Class Capture

See Note 1 at the end of this lesson.

2. What does the measure of a sector angle (or **central angle**) have to be in order for the sector to be 25% of the circle? 50%? 75%?

Answer: 90° , 180° , and 270°



Teacher Tip: Some students may find it challenging to get the angle measure to be exactly 90° , but it can be done with small, careful movements of the point. You may also take this opportunity to discuss inherent round-off error when the movement of points is constrained by pixels, regardless of whether students are able to get the angles to the desired measures. Students should come to realize that 25% of 360° is 90° , 50% of 360° is 180° , and so on. This also points to the connection that arc length and sector area are determined as a percentage of circumference and area.



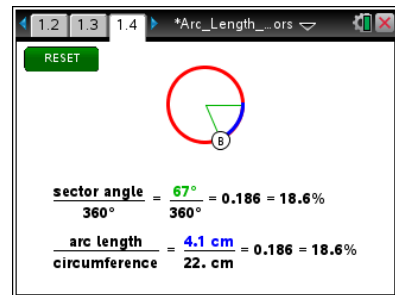
TI-Nspire Navigator Opportunity: *Quick Poll*

See Note 2 at the end of this lesson.

Move to page 1.4.

The central angle formed by a sector of a circle is a sector angle.

3. Drag point B around the given circle to test your previous answers. Were you correct?



Answer: Discuss student reactions.

4. Drag point B until the sector angle is 90° . Compare the ratio of the sector angle measure to 360° with the ratio of the arc length to the circumference. What do you observe?

Answer: The ratios are the same.

Teacher Tip: This is the underlying principle for determining arc length. Students need to understand that the key to finding arc length is that its length is a fraction (or percentage) of the total circumference. That fraction of the circumference is determined by the central angle. Central angle and sector angle are the same angle.

5. Would your answer to the previous question be different if the sector angle were a different measure?

Answer: No, the ratios would still be the same.

Teacher Tip: This is what you want students to see. These ratios are the same no matter what the central angle is.



6. How would you create an arc having a length that is approximately 33.3% of the circumference of the circle? Explain your reasoning.

Answer: You would have to make the central angle 120° . When the central angle is 120° , the ratio of the arc length to the circumference is approximately 33.3%.

Teacher Tip: In a traditional setting, students typically are given the formula for finding arc length, and then they are given a central angle and asked to find the resulting arc length. This question makes students think about the opposite. In other words, how would you get an arc length that is a certain fraction of the full circumference? It is intended to get them to think about the reasoning behind the formula. Emphasize this point with students.

7. What is the relationship between the length of the arc and the measure of the sector angle that intercepts the arc?

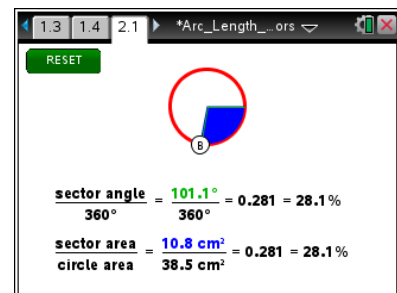
Answer: The measure of the sector angle determines the percentage of the circumference that defines the arc.

Teacher Tip: Take extra care to emphasize the relationship between the central angle and percentage. Students need to make the connection that as the central angle sweeps out the arc, the central angle itself is what determines the percentage of the whole circumference that is taken up by the arc. Then students will be able to conceptualize that the arc length is determined by multiplying a fraction (or percentage) times the circumference.

Move to page 2.1.

8. Drag point B around the given circle. How does the measure of the sector angle intercepting an arc relate to the area of its corresponding sector?

Answer: The measure of the sector angle determines the percentage of the area that defines the sector.



Teacher Tip: Again, take extra care to emphasize the relationship between the central angle and percentage. As in the previous problem, the central



angle determines the percentage of the whole area that is taken up by the sector. Then students will be able to conceptualize that sector area is determined by multiplying a fraction (or percentage) times the area.

9. How would you create a sector whose area is $\frac{2}{3}$ of the area of the circle? Explain your reasoning.

Answer: You would have to make the central angle 240° . When the central angle is 240° , the ratio of the area of the sector to the area of the circle is $\frac{2}{3}$, or approximately 66.7%.

Teacher Tip: Again, in a traditional setting, students are typically given the formula for finding the area of a sector and a central angle, and asked to find the resulting sector area. This question is also intended to get students to think about the reasoning behind the formula. Continue to emphasize this point.



TI-Nspire Navigator Opportunity: Quick Poll

See Note 3 at the end of this lesson.

10. What is the relationship between the measure of the sector angle and the area of the sector?

Answer: The measure of the central angle determines the percentage of the area defined by the sector.

Teacher Tip: Students need to make the connection that as the central angle sweeps through the circle, the central angle itself is what determines the percentage of the total area that is taken up by the sector. Then students will be better able to understand that the sector area is determined by multiplying a fraction (or percentage) times the area of the circle.

11. Describe in your own words the connection between arc length, area of a sector, and the ratio of the central (sector) angle to the circle.

Sample Answer: As a central angle changes, its corresponding ratio compared to 360° changes. The length of its intercepted arc and the area of its corresponding sector also change by the same ratio. That ratio can be used to calculate arc length and sector area as a fraction (or percentage) of the total circumference and total area of the circle.



Teacher Tip: Encourage discussion. It is important to have students verbalize this relationship. If they can understand and verbalize this concept, they will have a better understanding of the meaning of the formulas. Investigate the difference – and the connection – between the degree measure of an arc and arc length.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students:

- Recognize that the sum of two angles surrounding the center of a circle is 360° .
- Understand that the ratio of the central angle to 360° determines the length of an arc.
- Understand that the ratio of the central angle to 360° determines the area of a sector.



TI-Nspire™ Navigator™

Note 1

Question 1, Class Capture: Use Class Capture so that students can verify that the sum of the 2 angle measures is 360° for everyone in the class.

Note 2

Question 2, Quick Poll (Open Response): Use an Open Response Quick Poll to collect student answers to any of the percentages in question 2.

Note 3

Question 9, Quick Poll (Open Response): Have students send the measure of the central angle if you have a sector whose area is $\frac{1}{6}$ of the area of the circle. **Answer:** 60°