Logarithmic Transformations

MATH NSPIRED

TEACHER NOTES

Math Objectives

Students will change the parameters of the logarithmic function $y = a \cdot \log(b(x-h)) + k$ to discover that:

- Changes in *k* result in vertical translations.
- Changes in *h* result in horizontal translations.
- Changes in *a* result in vertical stretches and compressions (dilations), as well as reflections across the *x*-axis.
- Changes in *b* result in horizontal stretches and compressions (dilations), as well as reflections across the *y*-axis.
- Students will look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

- translation
- dilation
- compression
- stretch
- reflection
- transformation

About the Lesson

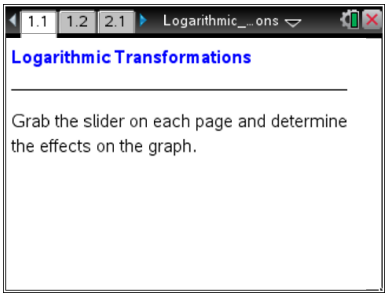
- This lesson involves graphing logarithmic functions of the form $y = a \cdot \log(b(x-h)) + k$.
- As a result, students will:
  - Manipulate given parameters and make conjectures about the relationships between the parameters' values and the effects on the graph of the resulting logarithmic function.
  - Test knowledge and determine the logarithmic function for a given graph.

TI-Nspire™ Navigator™

- Use Live Presenter for student demonstrations.
- Use Class Capture to examine patterns that emerge.
- Use Quick Polls to examine students' application of the learning.
- Use Teacher Software to review student documents.

Activity Materials

Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

Logarithmic_Transformations_Student.pdf

Logarithmic_Transformations_Student.doc

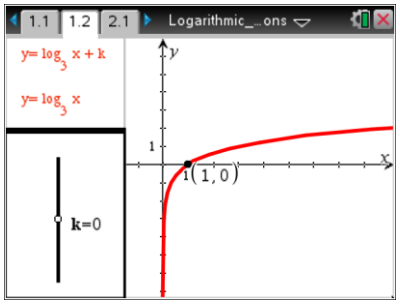
TI-Nspire document

Logarithmic_Transformations.tns

Discussion Points and Possible Answers

TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 1 at the end of this lesson.

Move to page 1.2.



1. For this activity, the function used is $y = a \cdot \log(b(x-h)) + k$.

   a. What effect does dragging the *k*-value have on the parent function $y = \log_3 x$? Change the *k*-value by grabbing and dragging the slider. What happens algebraically to the point (1, 0) in terms of *k* as the graph gets translated up or down?

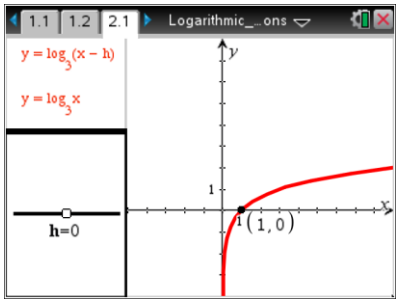
   Answer: Increasing *k*-values result in vertical translations up, and decreasing *k*-values result in vertical translations down. The point (1, 0) is translated to (1, *k*).

   b. Name the transformation (including its distance and direction) when the function $y = \log_3(x)$ changes to $y = \log_3(x) + 4$.

   Answer: A vertical translation up of 4 units. This is because 4 is added to every *y*-coordinate on the original function.

TI-Nspire Navigator Opportunity: *Quick Polls (Open Response)* and *Class Capture*
See Note 2 at the end of this lesson.

Move to page 2.1.



2. Change the *h*-value by grabbing and dragging the slider

   a. What happens to the equation and graph when $h < 0$?

   Answer: When the slider is moved to the left, the graph is translated to the left *h* units.

   b. Name the transformation (including its distance and direction) when the function $y = \log_3(x)$ changes to $y = \log_3(x-3)$.

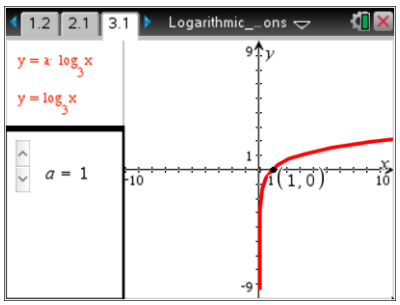
   Answer: A horizontal translation right of 3 units.

c. Chris says that the point (1, 0) on the parent function translates to (–3, 0) when she drags the *h*-value to –4 because the log of 1, base 3 is zero. Is her explanation mathematically correct? Explain. Change the *h*-value and confirm your explanation by grabbing and dragging the slider.

Answer: She is correct. $\log_3(-3--4)=\log_3(1)=0$

TI-Nspire Navigator Opportunity: *Quick Polls (Open Response)* and *Class Capture*
See Note 3 at the end of this lesson.

Move to page 3.1.



3. Change the *a*-value by clicking the arrows.
   a. As the *a*-value changes the graph, explain why the point (1, 0) remains on the transformed graph.

      Answer: When *x* = 1, for any base (non-zero and non-negative), any value of *a* multiplied by 0 is still 0, regardless of the value of *a.*

   b. When the graph $y=\log_3(x)$ is changed to $y=(-1/2)\cdot\log_3(x)$, what transformation has occurred? Describe the transformation in terms of what is happening with the points.

      Answer: Values greater than 1 result in vertical stretches, values between 1 and –1 result in vertical compressions, and negative values result in reflections over the *x*-axis.
      $y=(-1/2)\cdot\log_3(x)$ is a reflection over the *x*-axis and has been vertically compressed.

Teacher Tip: Ask students, "What happens when *a* =0?" The function is now *y* = 0, which lies on top of the *x*-axis.

Teacher Tip: A more general word to use for stretching and compressing is *dilation.* A dilation of scale factor ¼ is a compression, while a dilation of scale factor 4 is a stretch.

TI-Nspire Navigator Opportunity: *Quick Polls (Open Response)* and *Class Capture*
See Note 4 at the end of this lesson.

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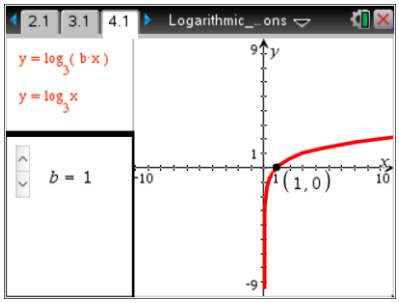

4. Change the *b*-value by clicking the arrows.
   a. When $b < 0$, what happens to the graph?

   Answer: If you click the slider until *b* is negative, the graph reflects across the *y*-axis. This is because negative *b*-values result in reflections over the *y*-axis.

   b. What other effects does the *b*-value have on the graph?

   Answer: Values greater than 1 result in horizontal compressions, values between 1 and –1 result in horizontal expansions, and negative values result in reflections over the *y*-axis.

   c. Suppose the function changes from $y = \log_3(x)$ to $y = \log_3(3x)$. Describe the transformation that occurs.

   Answer: The graph becomes horizontally compressed relative to the *y*-axis by a factor of 3.

> **Teacher Tip:** Ask students, "What happens when *b*=0?" Remind students that you can only take the logarithm of positive numbers.

TI-Nspire Navigator Opportunity: *Quick Polls (Open Response)* and *Class Capture*
See Note 5 at the end of this lesson.

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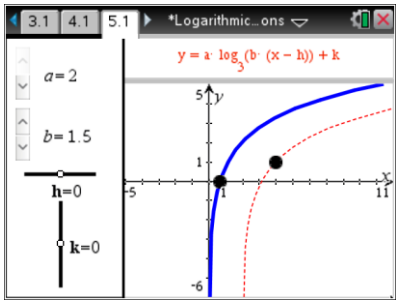

5. Apply what you have learned and change the values of variables *h*, *k*, *a*, and *b* by clicking their arrows so that the dashed graph is transformed to the solid graph in the displayed domain. It will say *correct* when you have done it correctly. Write the correct function here.

   Answer: $y = 2 \cdot \log_3(1.5(x-2)) - 1$ or $y = 2 \cdot \log_3(0.5(x-2)) + 1$

> **Teacher Tip:** Within this domain, there are two possible answers. This provides a great opportunity to challenge students to find both. Then have students explain why the two answers are equivalent.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 6 at the end of this lesson.

6. Nate says that transforming $y = \log_3(x)$ to $y = \log_3(x+2)$ is a horizontal translation of 2 to the right. Is Nate correct? Why or why not?

   Answer: Nate is incorrect. This is a common misconception. To obtain the correct answer, a horizontal translation of 2 units to the left, Nate should decide what *x*-value results in the expression *x* + 2 equaling zero.

7. What is the equation of the parent function $y = \log_3(x)$ translated 5 to the left and 2 up?

   Answer: $y = \log_3(x+5)+2$

8. a. Write the function that transforms $y = x^2$ with a horizontal translation to the right of 5 and a vertical dilation by a factor of 7.

   Answer: $y = 7(x-5)^2$

   b. Write the function that transforms $y = x$ with a vertical translation down 3 units.

   Answer: $y = x - 3$

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:
Given the exponential function $y = a \cdot \log(b(x-h))+k$:

- Changes in *k* result in vertical translations.
- Changes in *h* result in horizontal translations.
- Changes in *a* result in vertical stretches and compressions (dilations), as well as reflection across the *x*-axis.
- Changes in *b* result in horizontal stretches, compressions, and reflections across the *y*-axis.

Assessment

1. Name the transformation, and its distance and direction, when the function changes from **f**(x) = log(x) + 3 to **f**(x) = log(x) – 2.

   Answer: A vertical translation down 5 units.

2. Name the transformation, and its distance and direction, when the function changes from **f**(x) = log(x – 4) to **f**(x) = log(x + 2).

   Answer: A horizontal translation left 6 units.

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Note 1

Question 1, *Live Presenter*: This can be used to demonstrate technology techniques as well as the mathematical consequences of various actions.

Note 2

Question 1b, *Quick Polls* (*Open Response*) and *Class Capture*: Send an Open Response Quick Poll, asking students to submit their answers to question 1b. Check for understanding and discuss possible misunderstandings.

If students had difficulty, take a Class Capture of page 1.2. As a class, discuss the relationship between various k-values and the resulting graphs.

Note 3

Question 2b, *Quick Polls* (*Open Response*) and *Class Capture*: Send an Open Response Quick Poll, asking students to submit their answers to question 2b.

If students had difficulty, take a Class Capture of page 2.1 . As a class, discuss the relationship between various h-values and the resulting graphs.

Note 4

Question 3b, *Quick Polls* (*Open Response*) and *Class Capture*: Send an Open Response Quick Poll, asking students to submit their answers to question 3b.

If students had difficulty, take a Class Capture of page 3.1. As a class, discuss the relationship between various a-values and the resulting graphs.

Note 5

Question 4b, *Quick Polls* (*Open Response*): Send an Open Response Quick Poll, asking students to submit their answers to question 4b.

If students had difficulty, take a Class Capture of page 4.1. As a class, discuss the relationship between various *b*-values and the resulting graphs

Note 6

Question 5b, *Class Capture*: To observe students' progress on question 5b, take a series of Class Captures. Periodically discuss the results as a class.