Comparing Exponential and Power Functions

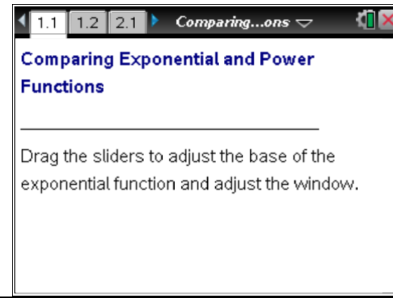
Student Activity

Name ______________

Class ______________

Open the TI-Nspire™ document Comparing_Exponential_and_Power_Functions.tns.

Which is greater for large values of x: an exponential function with a small base, or a power function with a large power? In this activity, you will use graphical representations to investigate this question.



Move to page 1.2.

1. Use the arrow labeled **b** to adjust the base of the exponential function to be 2, so you have the functions x^2 and 2^x.
   a. On what interval(s) is $x^2 < 2^x$? How do you know? Did you need the graph to determine this?

   b. Use the arrow to increase the base of the exponential function. For bases larger than 2, which is greater in the long run, b^x or x^2? How do you know?

2. Use the arrow labeled **b** to adjust the base of the exponential function to be less than 2.
   a. Are there any values of b for which the exponential function seems to be less than x^2 in the long run? What seems to be happening with these functions?

   b. Choose a value of b other than 1 so that x^2 appears to be greater than b^x in the long run. Now use the slider to change **xmax** in the graphing window. What appears to happen?

   c. Why couldn't you choose $b = 1$ in part 2b? What would have happened?

   d. Based on your observations, for any $b > 1$, what appears to be greater in the long run, b^x or x^2? Explain.

   e. How do you know that x^2 won't be greater than b^x at some point later in the graph?

   f. Do you think this result would be different if you were using the power function x^3 instead of x^2? Explain your thinking.

Move to page 2.1.

3. Use the arrow labeled **b** to change the base of the exponential.
   a. For what values of b does b^x appear to be greater than x^3 in the long run? What did you observe from the graph to lead you to this conclusion?

   b. Choose a small value of b (for example, 1.1) and use the **xmax** slider to adjust the window. What do you observe?

   c. For what values of b will b^x be greater than x^3 in the long run?

   d. Do you believe this result will hold true for all power functions? For what values of b and r do you think b^x will be greater than x^r in the long run?

Move to page 3.1.

4. Use the arrows to adjust the values of b and r. For what values of b and r does b^x appear to be greater than x^r in the long run? How does the graph help you to determine this?

Adjust b and r so that x^r appears to be greater than b^x for large values of x.

Move to page 3.2.

5. The function graphed on Page 3.2 is the ratio $\frac{b^x}{x^r}$ for the b and r you set on Page 3.1.

   a. For what values of $\frac{b^x}{x^r}$ is $b^x > x^r$? How do you know?

   b. What does the graph of $\frac{b^x}{x^r}$ tell you about the relationship between b^x and x^r for large values of x? Explain.

   c. Go back to Page 3.1, adjust the values of b and r, and return to Page 3.2 to observe the changes in the graph of the ratio. You might want to do this several times. Based on your observations, for what values of b and r will b^x be greater than x^r for large values of x? Explain your reasoning.