



Math Objectives

- Students will analyze the relationship between the motion of a particle along a straight line and the behavior of the graph of a general position function.
- Students will compare the steepness and direction of a position function to the velocity function.
- Students will visualize total area and signed area and relate these calculations to cumulative distance and position.
- Students will reason abstractly and quantitatively (CCSS Mathematical Practice).
- Students will look for and express regularity in repeated reasoning (CCSS Mathematical Practice).

Vocabulary

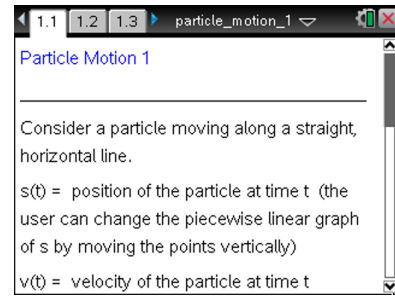
- position
- velocity
- magnitude
- cumulative distance
- signed area
- total area

About the Lesson

- This lesson involves the motion of a particle along a straight, horizontal line associated with a general position function.
- As a result students will:
 - Analyze the behavior of the graph of the position function as the particle moves to the left, right, and changes direction.
 - Compare the steepness of the graph of the position function to the velocity of the particle.
 - Compare and contrast total area and signed area, and relate these calculations to cumulative distance and position.
 - Use the graph of the position function to describe the general motion of a particle.
 - Create position functions which satisfy given conditions.

TI-Nspire™ Navigator™ System

- Transfer a File
- Use Screen Capture to examine patterns that emerge
- Use Live Presenter to demonstrate
- Use Teacher Edition computer software to review student documents
- Use Quick Poll to assess students' understanding



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing **ctrl** **G**.

Lesson Files:

Student Activity
Particle_Motion_2_Student.pdf
Particle_Motion_2_Student.doc
TI-Nspire document
Particle_Motion_2.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.



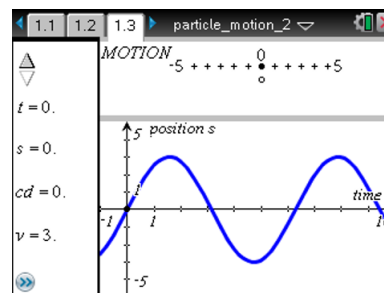
Discussion Points and Possible Answers

Tech Tip: On page 1.2, use the function `reset()` to set the time to 0. To define the position function on page 1.2, use the calculator command `Define` or by using “:=” (the assignment characters). For example, $pos(t) := t \cdot \cos(t)$.

- For t measured in seconds, $s(t)$ is the position of the particle at time t , $v(t)$ is the velocity of the particle at time t , and $cd(t)$ is the cumulative distance traveled by the particle from time $t = 0$.
- The values of t , s , cd , and v are given in the left panel on Page 1.3 and Page 1.4.
- Use the clicker arrows to change the value of t . On Page 1.3, you can also grab and move the open circle on the horizontal axis to change the value of t .
- The position graph s is displayed on page 1.3, and the velocity graph v is shown on page 1.4.
- The motion of the particle is modeled in the top panel on both pages.
- The function s can be redefined on page 1.2.

Move to page 1.3.

1. Use the clicker arrows to change the value of t . You can also drag the position slider (the open circle) attached to the horizontal axis (the t -axis) to change the value of t . Observe the position of the particle, the cumulative distance traveled, and the velocity of the particle.
 - a. Describe the behavior of the position graph s when the particle is moving to the right.



Answer: The position graph s is rising, or increasing, whenever the particle is moving to the right.

- b. Describe the behavior of the position graph s when the particle is moving to the left.

Answer: The position graph s is falling, or decreasing, whenever the particle is moving to the left.



- c. Describe the behavior of the position graph s when the particle changes directions.

Answer: The position graph s tends to level off, or flatten out, whenever the particle changes directions.

2. Compare the graph of the position function s on page 1.3 and the graph of the velocity function v on page 1.4.

- a. As t increases from 0 to approximately 1.6, describe how the steepness changes in the position graph s and how the magnitude of the velocity changes.

Answer: The position graph s becomes less steep as t increases. The magnitude of the velocity becomes smaller and decreases to 0.

- b. As t increases from approximately 1.6 to 4.7, describe how the steepness changes in the position graph s and how the magnitude of the velocity changes.

Answer: The position graph s becomes more steep until it reaches a maximum steepness at approximately $t = 3.1$ and then becomes less steep as t approaches 4.7. The magnitude of the velocity starts out small and increases until approximately $t = 3.1$ and decreases as t approaches 4.7.

Teacher Tip: Students may also notice that the graph of the position function appears to be fairly linear from $t = 2.7$ seconds to $t = 3.5$ seconds. In addition, the velocity is changing very slowly during that time interval.

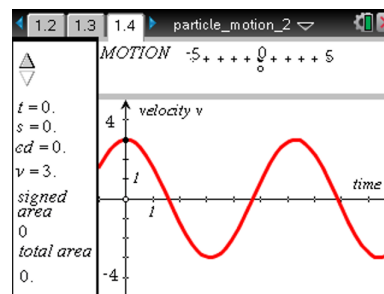
- c. Consider values for t greater than 4.7, and compare the steepness of the position graph s and the relationship to the velocity. Make a general conjecture about the steepness of the position graph s and the velocity function.

Answer: The magnitude of the velocity appears to measure the steepness of the graph of the position function s . Note that the velocity is positive whenever s is rising, or increasing, and negative whenever s is falling, or decreasing.



Move to page 1.4.

3. Use the clicker arrows to change the value of t . On page 1.4, you cannot grab and move the point on the horizontal axis. The shaded part of this graph is the region bounded by the velocity graph v , the horizontal axis, and the vertical lines at 0 and t . In the left panel, the total area is the area of all shaded regions. The signed area is the sum of all the areas above the horizontal axis minus the areas below the horizontal axis.
- Compare the signed area (computed and displayed in the left panel) and the position of the particle.



Answer: The signed area and the position of the particle are the same for all values of t .

- Compare the cumulative distance traveled and the total area (computed and displayed in the left panel) of the shaded region.

Answer: The total area and the cumulative distance are the same for all values of t .

4. On page 1.2, reset the time to 0, and define the position function to be $\ln(t + 1)$. To define the position function on page 1.2, use the calculator command Define or by using “:=” (the assignment characters). For example, $pos(t) := t \cos(t)$.
- Use the resulting graph of the position function on page 1.3 to describe the motion of the particle, the velocity, and the total distance traveled over the time interval $(0,10)$.

Answer: The particle starts at 0 and is always moving to the right. The velocity is always positive. Since the position graph becomes less steep as t increases, the velocity is decreasing.

- Why is the position equal to the cumulative distance over the interval $(0,10)$?

Answer: The particle starts at 0 as is always moving to the right. Therefore, the velocity is always positive and there is no area bounded by the velocity graph v below the horizontal axis.

Teacher Tip: Some students might choose to analyze the velocity graph on page 1.4 to answer this question.

Tech Tip: Use the calculator key e^x for exponentiation, not the letter E .



5. On page 1.2, reset the time to 0, and define the position function to be $3e^{-t/4}$.
- Use the resulting graph of the position function on page 1.3 to describe the motion of the particle, the velocity, and the total distance traveled.

Answer: The particle starts at 3 and is always moving to the left, approaching 0. The velocity is always negative. As t increases, the velocity also becomes smaller in magnitude (the position graph s becomes less steep), closer to 0.

- How would you use the area of the shaded region on page 1.4 to compute the position of the particle at any time?

Answer: The entire shaded region is below the horizontal axis. Therefore, subtract the area of the shaded region from the initial position, 3, to find the position of the particle at any time. This is equivalent to adding the signed area to the initial position.

TI-Nspire Navigator Opportunity: Screen Capture and Live Presenter

See Note 1 at the end of this lesson.

Tech Tip: To adjust the window settings, select menu > Window /Zoom > Window Settings.

6. On page 1.2, reset the time to 0. In each part below, try to define a position function that satisfies the given properties over the time interval $(0,10)$.
- The position of the particle is 0 at times $t = 0, 3,$ and 7 .

Sample Answers: Answers will vary. One possibility is

$$pos(t) = \frac{1}{10} t(t-3)(t-7) = \frac{1}{10} (t^3 - 10t^2 + 21t).$$

- The position of the particle is -4 at time $t = 0$. The particle has positive velocity until time $t = 5$ and negative velocity after time $t = 5$.

Sample Answers: Answers will vary. Consider the following:

$$pos(t) = -4 \cos \left(\frac{\pi t}{5} \right).$$



- c. The position of the particle is 4 at time $t = 0$. The position of the particle oscillates about 0, but with smaller oscillations as time goes on.

Sample Answers: Answers will vary. One possibility is

$$pos(t) = 4e^{-t/4} \cos(2t).$$

Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to understand:

- The relationship between the motion of a particle along a straight line and the graph of the position function.
- The relationship between the motion of a particle along a straight line and the graph of the velocity function.
- The relationship between the steepness of a position function and the magnitude of the velocity function.
- How signed area associated with the graph of the velocity function can be used to calculate the position of the particle.
- How total area associated with the graph of the velocity function can be used to calculate the cumulative distance traveled by the particle.

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Note 1

Question 6, Name of Feature: Screen Capture and Live Presenter

Use Screen Capture and Live Presenter with your class to consider various answers to Question 6.