Triangle Area

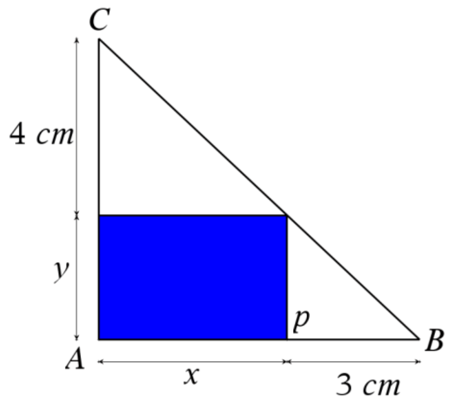
Teacher Notes & Answers

7 8 9 10 **11** **12**

TI-Nspire™ Investigation Student 20 min

Introduction

A right-angled triangle contains a rectangle. The base of the triangle is equal to the length of the rectangle (x) plus an additional 3cm in length. The height of the triangle is equal to the height of the rectangle (y) plus an additional 4cm in length.



In this activity we are first interested in the area of the rectangle, then the area of the triangle which circumscribes the rectangle.

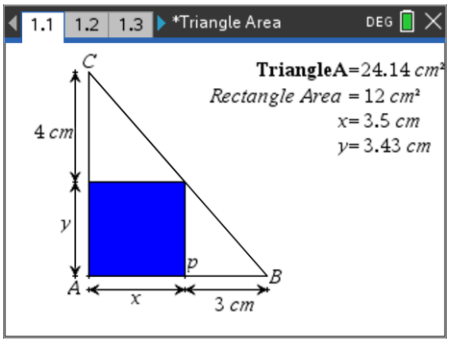
The TI-Nspire file: Triangle Area contains a dynamic representation of the setup shown opposite.

Exploring

Open the TI-Nspire document: Triangle Area

Point P is dynamic and can be used to change the shape of the rectangle and the circumscribed triangle such that the triangle will continue to follow the properties outlined in the introduction.

Drag point P and observe what happens to the area of the rectangle and the area of the triangle.



Question: 1

What do you notice about the area of the rectangle? Use your geometry knowledge to prove the result.

Answer: The rectangle has a constant area.

Let Q be the point where the rectangle touches the hypotenuse of triangle ABC and point R be where the rectangle touches the height.

Triangle BPQ and QRC are similar, (ASA) therefore: $\frac{4}{x} = \frac{y}{3}$, transposing gives: $xy = 12$, area is constant.

Question: 2

Determine an expression for the area of the triangle in terms of x and y.

Answer: $Area = \frac{1}{2}(x+3)(y+4)$

Question: 3

Use your results from Question 1 and 2 to express the area of the triangle in terms of x only.

Answer: From Question 1 we know that $y = \frac{12}{x}$, therefore triangle area is: $Area = \frac{1}{2}(x+3)\left(\frac{12}{x}+4\right)$

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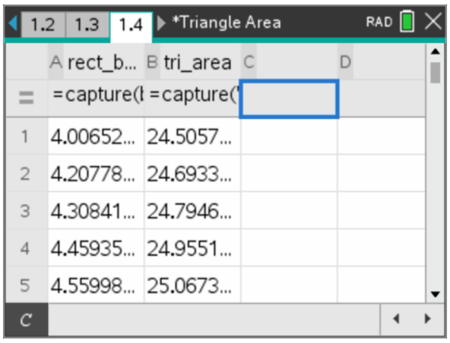
Whenever you move point P, data for the area of the triangle and base of the rectangle (x) is automatically captured on page 1.4 of this document.

Sample shown opposite.

The variables are called: **rect_base** (x) and **tri_area** (triangle area).

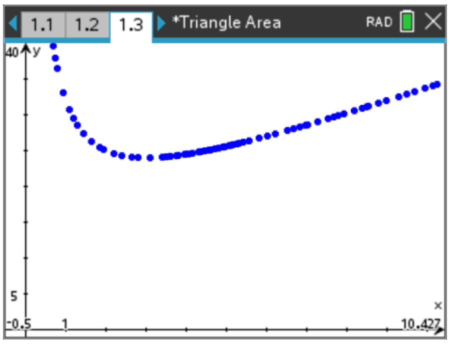
If page 1.4 does not contain many data values (or none at all), drag point P on page 1.1 around to populate the spreadsheet.



Navigate to page 1.3 (Graph Application).

The data for the triangle area and base measurement should be displayed as a scatterplot.

Graph your function from Question 3 to make sure it passes through the data.



Question: 4

Use calculus to determine the minimum area of the triangle.

Answer: $a(x)=\frac{1}{2}(x+3)\left(\frac{12}{x}+4\right)$. Note that x > 0.

$$a'(x)=2-\frac{18}{x^2}$$

$$a'(x)=0 \ when \ x=3$$

Question: 5

Maximum and minimum area problems associated with geometry often occur for 'regular' shapes. For example; the maximum area for a rectangle with fixed perimeter occurs when the rectangle is a square. It is possible for triangle ABC to be 'half a square' if x = 4 and y = 3. Adjust point P to see that this is true, then move point P to the point where the area of triangle ABC is a minimum and explore what is happening at that point.

Answer: The minimum area occurs when triangles BPQ and QRC are congruent (equal). Remember that the circumscribed rectangle has a fixed area, so the triangle has restrictions applied. In some sense this is similar to the typical rectangle / area problem where one side of the rectangle is not included. (Boarders a river etc.) In these types of problems the side that is not included in the calculation biases the result.

In this case if we remove the area of the 'fixed' rectangle, that is subtract 12 from the area of triangle ABC, then the minimum occurs when at the same point, where triangles BPQ and QRC are equal. (congruent)

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