Sample some Pi Monte

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Student Activity

7   8   9   **10   11   12**

TI-Nspire   Investigation   Student   45 min

Introduction

The Monte-Carlo technique uses probability to model or forecast scenarios. In this activity the Monte Carlo technique will be used to estimate an area; however the learning focus is on the logic and understanding rather than the single numerical result. The numerical result simply verifies that the processes work.

Creating the Simulation

Start a **new document** and insert a **Graph application**.

Use the **[Menu]** to change the graph type:

> **Graph Entry/Edit > Relation**

Graph the relation:

$$x^2 + y^2 = 1$$

Type in the equation then press **[Enter]** to graph it.

The graph is the focus so it is useful to zoom in a little so that it fills the screen a little more.

> **[Menu] > Window/Zoom > Zoom In**

The mouse now appears as a magnifying glass. Place it as close as possible to the origin and click. Click again. Three clicks will zoom too much. If you inadvertently zoom too much you can select zoom out or press **Ctrl + Z** (undo).

The next task is to create two lists. A spreadsheet is a great place to work with lists. Press **Ctrl + I** and insert a spreadsheet.

Navigate to the top of the spreadsheet (shown opposite) and type in the names for the lists: **xp** and **yp**.
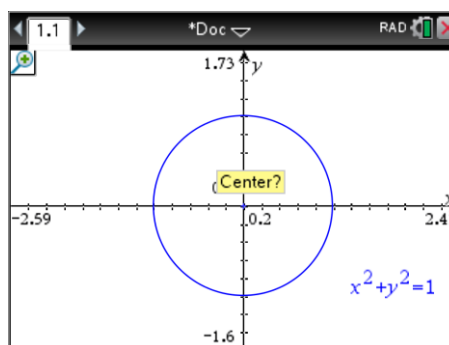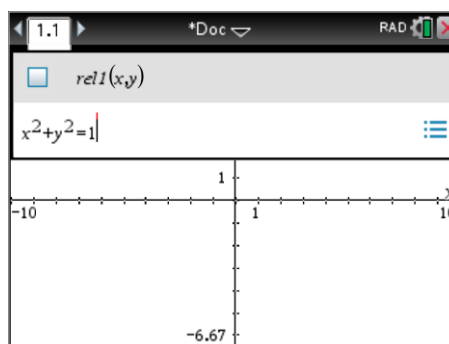
The TI-Nspire CX contains lots of keyboard short cuts. Ctrl + I = Insert; Ctrl + C = Copy. You may be able to guess a lot more. Check out our Short Cuts document in the Getting Started resources.

Keys can be pressed successively rather than simultaneously to access short cuts.

TEXAS INSTRUMENTS

Navigate to cell A1. Cells are referenced the same as any other spreadsheet application, similarly formulas start with an 'equals' sign.

In cell A1 type the formula:

= 2 rand() – 1

A random number will be generated[1], notice however that the formula used to generate the random number is visible at the bottom of the screen. Press Ctrl + R and notice what happens to the random number.
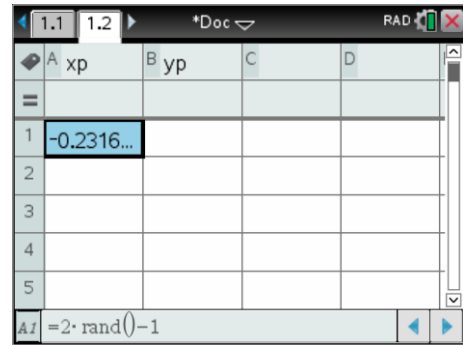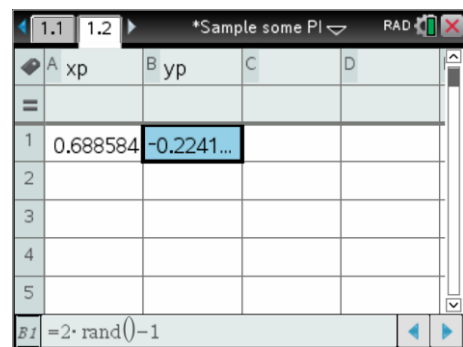
Question: 1.

Rand() generates random numbers between 0 and 1. What sort of numbers does: 2rand() – 1 generate?

The formula also needs to be entered into cell B1. This can be done by retyping the expression, preceded again by the '=' sign.

Alternatively, while cell A1 is highlighted press Ctrl + C (Copy), navigate to cell B1 and press Ctrl + V. (Paste).
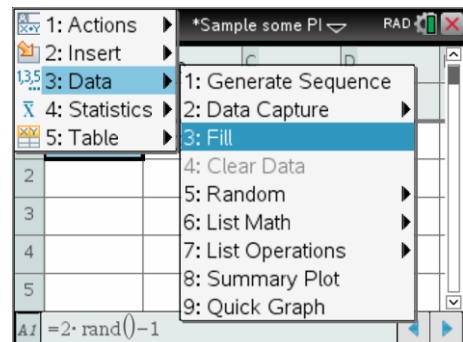
The same formula needs to be entered in all the cells from A1 to A25. Rather than typing or copying a quicker approach is available.

Navigate to cell A1. With cell A1 highlighted press:

[Menu] > Data > Fill

Arrow down to cell A25 (notice that a dotted line flows down) … press **[Enter]** to generate 25 random numbers.
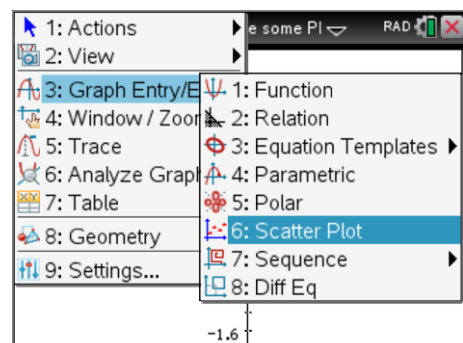
Repeat this process for the second column: B1 to B25.

The random points need to be plotted around the circle in the Graph application. Navigate back to the Graph application by pressing Ctrl + Left (Navigation pad) or by clicking on the page tab.

Use the menu to change the graph type to a Scatter Plot.

[Menu] > Graph Entry/Edit > Scatterplot

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[1] **Random Numbers**: Electronic devices generate pseudo random numbers. If your 'random' number is the same as anyone else's you should change your 'seed' value. In a calculator application type: randseed #### and enter a four digits (#) number.

S1 = Scatterplot One

The list names can be typed in directly, alternatively press the **[VAR]** key and select the desired list.

The numbers are random so technically it doesn't matter which list is assigned to which axis, however for consistence of labelling, put the 'xp' list on the x axis and the 'yp' list on the y axis.

There are 25 points plotted on the graph opposite. We are interested in what proportion of points land inside the circle. In the image opposite there are 9 points that fall outside the circle, therefore 16 points inside. The proportion of points inside the circle is therefore:

$$\frac{16}{25} = 0.64$$

To generate another sample, return to the spreadsheet (Ctrl + Right) and press Ctrl + R to generate another 25 random points. Navigate back to the graph and determine the proportion of points inside the circle.

In the screen shown opposite one point is clearly outside the circle but there are two others that appear to be right on the line. You can continually zoom in on these points and the zoom out or 'undo' the zooming. Decide an appropriate rule to declare points on the line as either in / out.

The screen show opposite shows how close points can be to falling 'on the line' even after zooming several times.
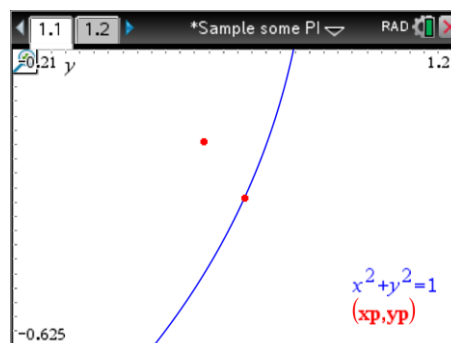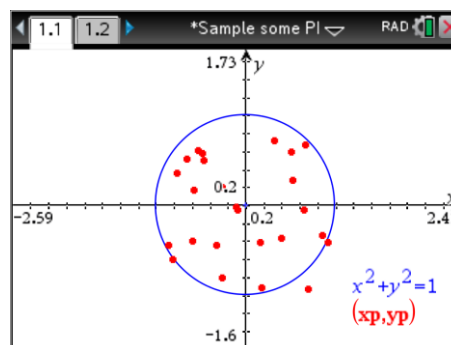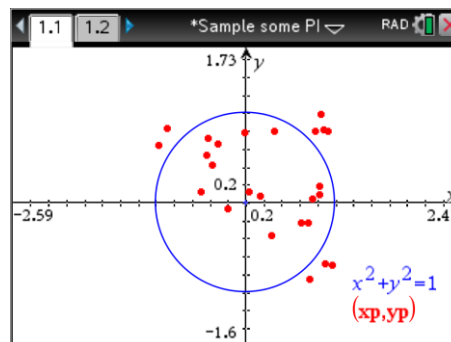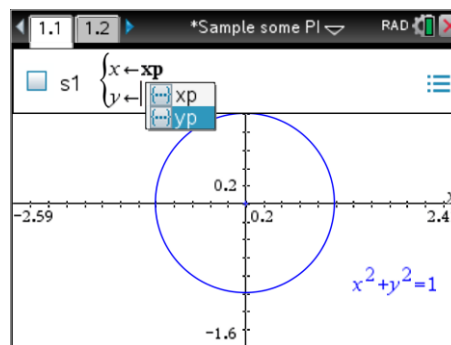
Question: 2.

Generate 10 samples each of size 25 and record the proportion of points that land inside the circle in each trail.

Trial:	1	2	3	4	5	6	7	8	9	10
Proportion:										

Question: 3.

Based on your data so far, estimate the proportion of points that land inside the circle.

In the spreadsheet application label column D: "Data".

Record the proportions of points that landed inside the circle for each of your ten trials.

Collect the results from 9 of your friends and add them to the "Data" column for a total of 100 results.
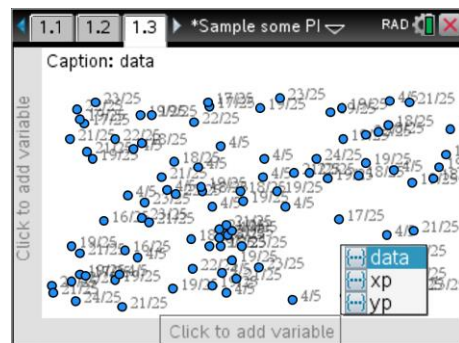
Once all the results have been entered, press Ctrl + I and insert a Data and Statistics application. The points appear randomly scattered around the screen.
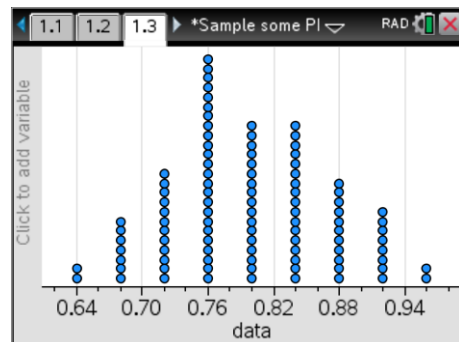
Press [Tab] to bring up the variable list for the x axis.

(Or click on 'add variable)

Select 'data'

The graph displays the distribution of the proportions, referred to as: 'the sampling distribution'.

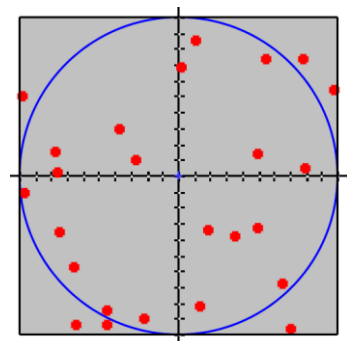## Question: 4.

Determine the mean of your sampling distribution.

Question: 5.

Describe the spread of the results using words like 'most'.

Question: 6.

If you generated more samples would the data mean or spread (or both) change? Discuss.

The randomly generated points all fit inside a square. The proportion of points landing inside the circle represents the proportion of the square's area occupied by the circle.

Question: 7.

Given the circle has a radius of 1 unit, determine the area of the square and the area of the circle according to the mean of the sampling distribution.

TEXAS INSTRUMENTS

Extension

If 100 random points were generated it would be much harder, and more time consuming to count the points that fall either inside or outside the circle. It is possible to get the calculator to count them automatically and keep a tally of the results!

The first task is to generate more points. Use the FILL command to populate cells A1 through to B100 with the formula:

=2 rand() – 1

Navigate to the top of column C. Enter the title for the list:

dist

This column will measure the distance between the origin and each random point. Pythagoras's theorem can be used to compute this distance. In the formula cell, immediately below the column label, enter the formula:

$$= \sqrt{xp^2 + yp^2}$$

All 100 distances will be automatically computed, even when the next set of random points is generated. From here on each sample contains 100 points.

Navigate to cell E1. This cell will count all the points that land inside the circle. In cell E1 type the formula:

= countif (dist , ? < 1) / dim(dist)

The 'countif' command is common in most spreadsheet applications. The formula counts the elements in the specified list ('dist') that satisfy the condition '< 1'. Therefore all distances less than 1 will be counted, in other words all the points inside the circle. The result is then divided by the number of points, the dimensions (dim) of the distance (dist) list.

Press **Ctrl + R** a couple of times to observe the result.

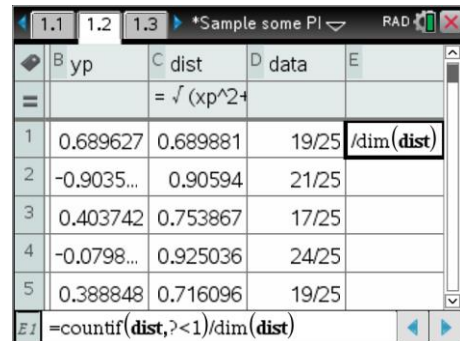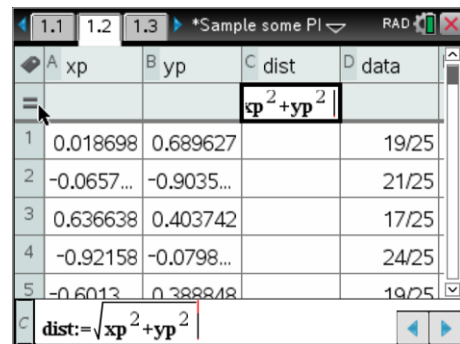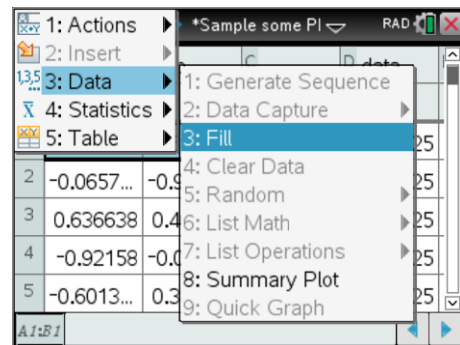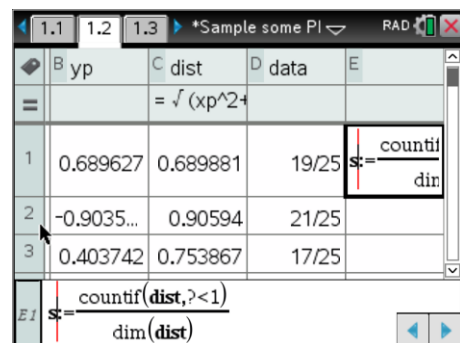**Ctrl + =** for "<, >, ≤, ≥" and [?!▸] for "?"

Select cell E1 then press:

Ctrl + [Var]

Store the calculation as:

S   (Abbreviation for Sample)

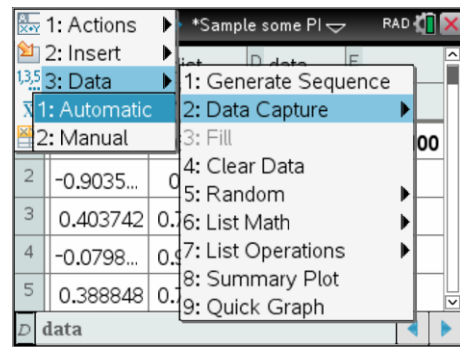The sample result will automatically be stored in "S".

TEXAS INSTRUMENTS

Navigate to the formula bar in the data column. This column will now be used to automatically capture the data. Press:

[Menu] > Data > Capture > Automatic

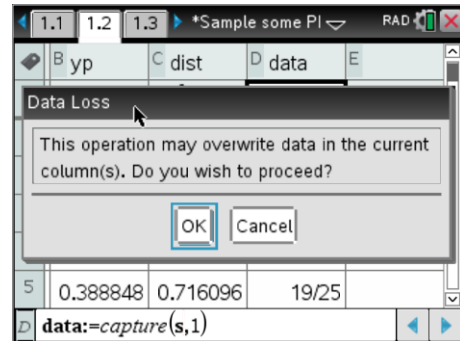Enter "**S**" for the requested variable.

A warning message will appear advising you that all existing data will be erased, press [Enter] to accept the message.

The data column will now contain just one sample result, the result from the current set of calculations.

Press Ctrl + R

What happens each time Ctrl + R is pressed?

If you would like to keep track of how many samples you have generated, navigate to cell E2 and enter the formula:

=dim(data)

Each time a new sample is generated the 'countif' command will count how many points appear inside the circle, the result is automatically captured and added to the data list. The dim command then counts the number of entries in the data list.

Continue pressing Ctrl + R until 100 samples have been generated, remember this is 100 samples each one representing the proportion of 100 points that landed inside the circle!
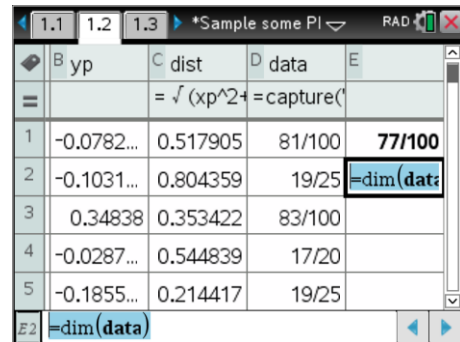
Question: 8.

Return to the data and statistics application. This will now contain the data for 100 samples, each of 100 points. Determine the mean of your sampling distribution.

Question: 9.

Describe the spread of the results using words like 'most'.

Question: 10.

Compare the results from this sampling distribution and your original one where each sample consisted of 25 points. Discuss the results and reflect upon your answer to question 6. Note that you can also continue generating samples by pressing Ctrl + R and noting any changes to the mean or spread of the sampling distribution.