# Exponential Reflections

TEACHER NOTES

## About the Lesson

In this activity, students will investigate the inverse of an exponential function by observing a scatterplot. Students will determine that the inverse of an exponential function is a logarithmic function. As a result, students will:

- Analyze the function $f(x) = 2^x$, its corresponding inverse function $g(x) = log_2 x$, and their reflection about the line $y = x$.
- Analyze the function $f(x) = e^x$, its corresponding inverse function $g(x) = lnx$, and their reflection about the line $y = x$.
- Graph the function $f(x) = 10^x$ and its corresponding inverse function $g(x) = logx$.

## Vocabulary

- line reflections
- symmetry
- logarithmic function
- exponential function
- inverse functions

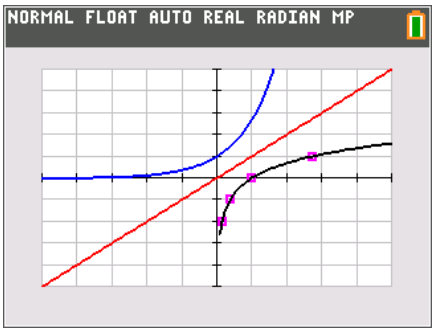
## Teacher Preparation and Notes

- Students should be familiar with the concept of inverse functions.

## Activity Materials

- Compatible TI Technologies: TI-84 Plus*, TI-84 Plus Silver Edition*, TI-84 Plus C Silver Edition, TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



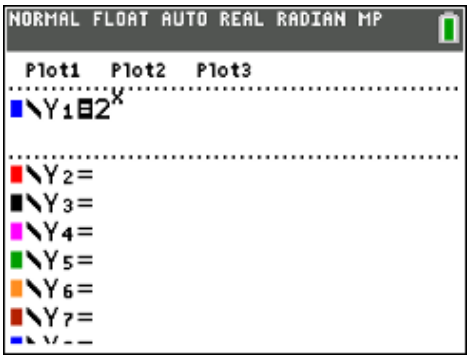

**Tech Tips:**

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

**Lesson Files:**

- Exponential_Reflections_84_Student.pdf
- Exponential_Reflections_84_Student.doc

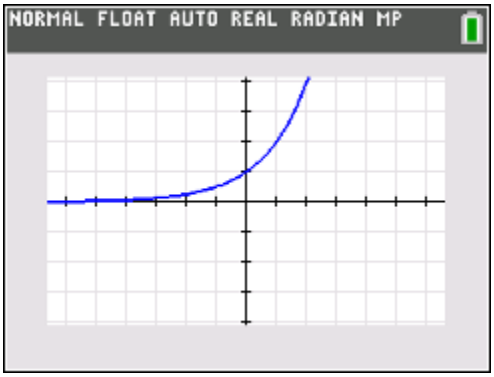
In this activity, you will investigate the inverse of an exponential function. You will also investigate the symmetry of the exponential function and its inverse.



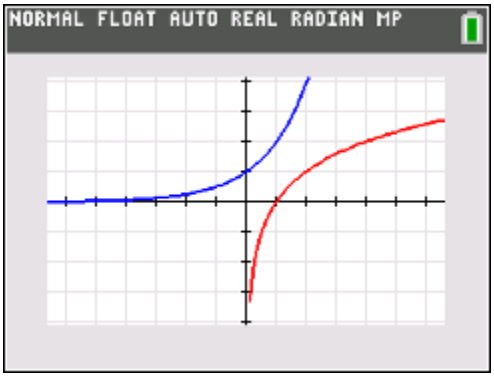

**Problem 1 –** Reflecting an Exponential Function

1. Enter the exponential function $f(x) = 2^x$ on the [y=] screen. Press [zoom] and select 4: ZDecimal.

   A function is invertible if each output value is mapped from a unique input value. Is the function $f(x) = 2^x$ invertible? What would the inverse of this graph look like? Sketch the function $y = 2^x$ and its inverse on the grid to the right.




   **Answers**: Yes, the function $f(x) = 2^x$ invertible. Since the graph of the function $f(x) = 2^x$ is increasing, concave up and has a horizontal asymptote of $y = 0$, the inverse graph would be increasing, concave down, and have a vertical asymptote of $x = 0$. Students may also notice that the inverse passes through (1, 0) and has a domain of $(0,\infty)$ and a range of $(-\infty\infty)$. The sketch drawn by the students should be similar to the calculator screen shot to the right.




**Tech Tip:** If your students are using the TI-84 Plus CE have them turn on the GridLine by pressing [2nd] [format] to change the graph settings. If your students are using TI-84 Plus, they could use GridDot.

2. Press [2nd] [table] to access a table of values for your function.

   Record the *y*-values under the original *y*-value column in the table below. Recall that if the function $f(x) = 2^x$ consists of input-output pairs $(a, b)$, then the inverse function consists of input-output pairs $(b, a)$. Record the inverses of each point by switching the *x*- and *y*-values and recording the results in the inverse columns in the table below.

**Answers**:

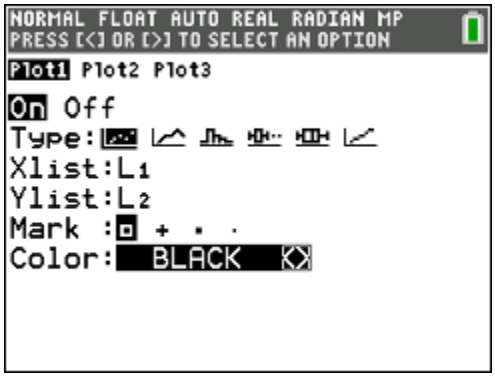
| Original *x*-value | Original *y*-value | Inverse *x*-value | Inverse *y*-value |
|---|---|---|---|
| **–2** | 0.25 | 0.25 | –2 |
| **–1** | 0.5 | 0.5 | –1 |
| **0** | 1 | 1 | 0 |
| **1** | 2 | 2 | 1 |
| **2** | 4 | 4 | 2 |
| **3** | 8 | 8 | 3 |

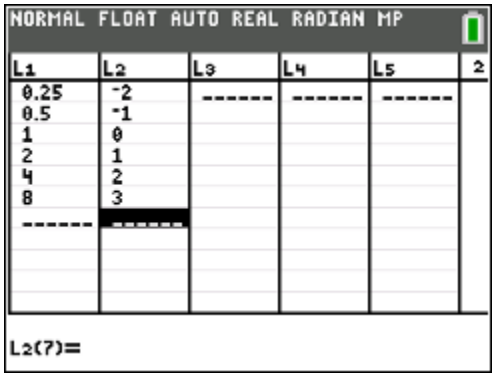
3. Plot these inverse points by pressing [stat] and selecting 1: Edit. Enter the inverse values in **L1** and **L2**.

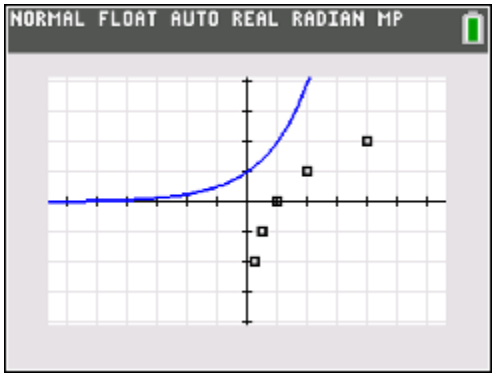
   To set up the scatter plot of the two lists, press [2nd] [stat plot] and match the screen to the right. Now press [graph] to observe the plotted values.

   Do your plotted points appear to be on the graph of the inverse function that you sketched in Question 1?

   **Answers**: The plotted points should appear to be on the graph sketched by the students in Question 1.



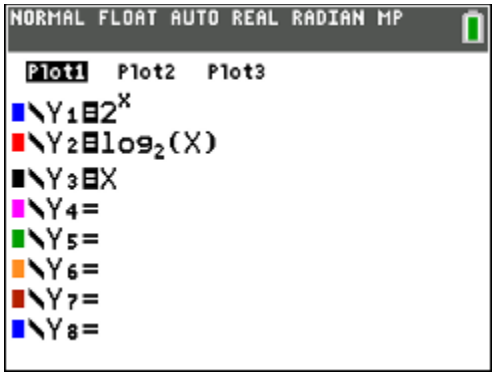
4. The inverse of a general exponential function $f(x) = b^x$ is a logarithmic function of the form $g(x) = log_b x$. Write the inverse function of $f(x) = 2^x$.

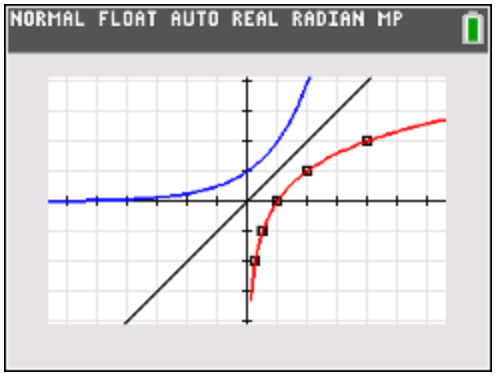
   **Answer:** $g(x) = log_2 x$ or $f^{-1}(x) = log_2 x$

5. Check your result by graphing this function in Y2 to see if it passes through all the plotted points. Also graph the identity function Y3 $= x$. Are the two graphs symmetric with respect to the line $y = x$?

   **Note:** The $log_b x$ is found by pressing [math] and A: logBASE(.

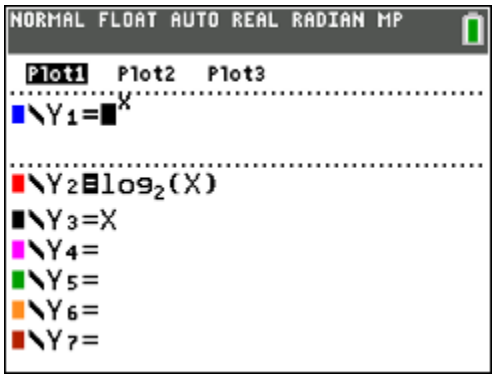
   **Answer:** Yes, the graphs are symmetric with respect to the line $y = x$. The graphs appear to be reflected across the line $y = x$.

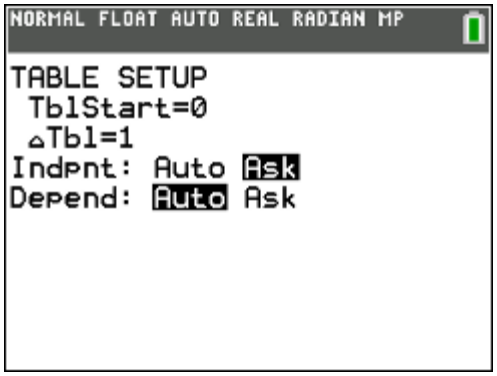


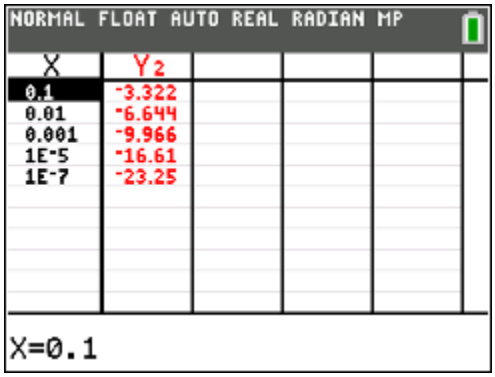




**Teacher Tip:** Students may notice that the graph of the logarithmic function appears to stop as the graph approaches the *y*-axis (as *x* approaches 0 from the right.). This is a great opportunity to explore a table to convince the students that the graph does not stop. See **Optional Notes.**

**Optional Notes:** Go to [y=] and deselect Y1 and Y3 so that students will only see the table values for the logarithmic function. Select [2nd] [tablset]. Change Indpnt: to Ask. Select [2nd] [table] and explore values that approach 0 from the right.



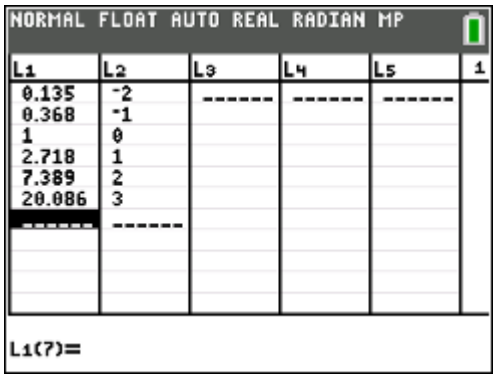

**Problem 2 –** The inverse of $f(x) = e^x$. This function has a natural base of *e*.

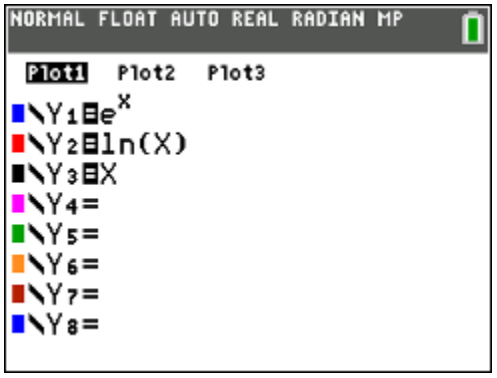
6. Graph *Y1* = $e^x$. Repeat the steps in **Problem 1** using $f(x) = e^x$.

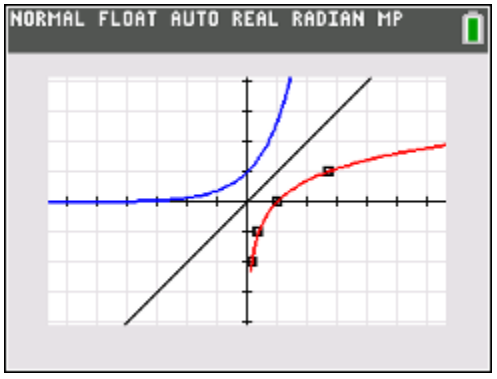
   What is the inverse function of $f(x) = e^x$?

   **Note:** The inverse of $f(x) = e^x$ is called a Natural Logarithmic function.

   **Answer:** $g(x) = lnx$ or $f^{-1}(x) = lnx$




**Teacher Tip:** Students will likely write $g(x) = log_e x$ and may use this notation to graph the logarithmic function. This is a good time to have students notice the relationship on the keypad of the [ln] key and [2nd] [$e^x$] and that $g(x) = log_e x$ should be written as $g(x) = lnx$.

**Teacher Note:** These are the table values if the students make a table while completing **Problem 2.**

| Original *x*-value | Original *y*-value | Inverse *x*-value | Inverse *y*-value |
|---|---|---|---|
| **–2** | 0.1353 | 0.1353 | –2 |
| **–1** | 0.3679 | 0.3679 | –1 |
| **0** | 1 | 1 | 0 |

| | | | |
|---|---|---|---|
| **1** | 2.7183 | 2.7183 | 1 |
| **2** | 7.3891 | 7.3891 | 2 |
| **3** | 20.086 | 20.086 | 3 |

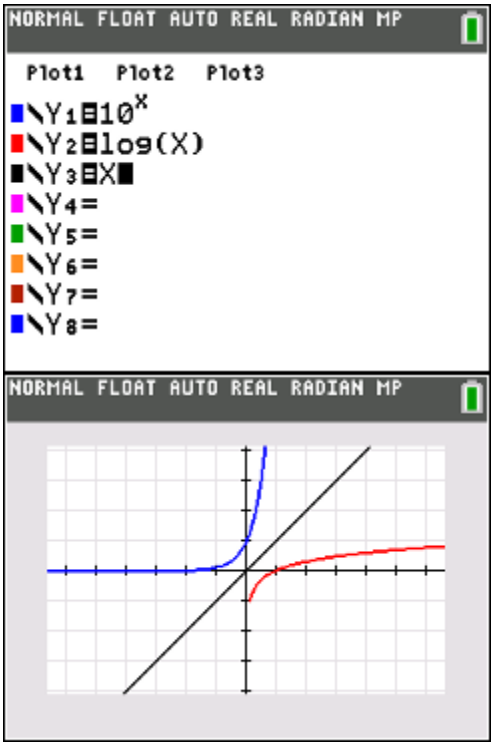
**Problem 3 –** The inverse of $f(x) = 10^x$.

7. Graph *Y1* = $10^x$

   Find the inverse function of $f(x) = 10^x$. Check the symmetry of the function and its inverse by graphing.

   **Note:** The inverse of $f(x) = 10^x$ is called a Common Logarithmic function.

   **Answer:** $g(x) = logx$ or $f^{-1}(x) = logx$




**Teacher Tip:** Students will likely write $g(x) = log_{10}x$ and may use this notation to graph the logarithmic function. This is a good time to have students notice the relationship on the keypad of the [log] key and [2nd] [$10^x$] and that $g(x) = log_{10}x$ should be written as $g(x) = logx$.