


TI-84 PLUS CE FAMILY
Math Objectives

- Students will use the unit circle and the handheld to estimate the six trig functions at certain angle measures.
- Students will then find patterns in the results of the six trig function estimations and discuss them with their classmates.
- Students will try to make a connection with how to understand these topics in IB Mathematics courses and on their final assessments.

Vocabulary

- Unit Circle
- Right Triangle Trigonometry
- Radian Measure

About the Lesson

- This lesson is aligning with the curriculum of IB Mathematics Applications and Interpretations HL and IB Mathematics Approaches and Analysis SL/HL
- This falls under the IB Mathematics Content Topic 3 Geometry and Trigonometry:

AI HL 3.7: (a) The definition of a radian and conversion between degree and radian

3.8: (a) The definition of $\cos \theta$ and $\sin \theta$ in terms of the unit circle

AA SL/HL 3.4: The circle: radian measures of angles

3.5: (a) Definition of $\sin \theta$, $\cos \theta$ in terms of the unit circle

(b) Definition of $\tan \theta$ as $\frac{\sin \theta}{\cos \theta}$

(c) Exact values of trig ratios of $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ and their multiples

3.9: (a) Definition of the reciprocal trig ratios $\sec \theta$, $\csc \theta$, and $\cot \theta$

As a result, students will:

- Apply this information to real world situations.

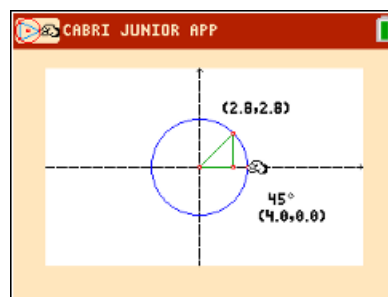
Teacher Preparation and Notes.

- This activity is done with the use of the TI-84 family as an aid to the problems.

Activity Materials

- Compatible TI Technologies: TI-84 Plus*, TI-84 Plus Silver Edition*, TI-84 Plus C Silver Edition, TI-84 Plus CE

* with the latest operating system (2.55MP) featuring MathPrint™ functionality.


Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at <http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials>

Lesson Files:

Student Activity

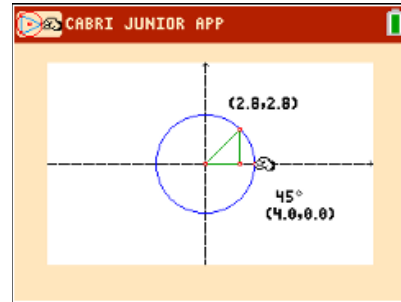
84CE-TrigPatterns-Student.pdf

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UNITC.8xv


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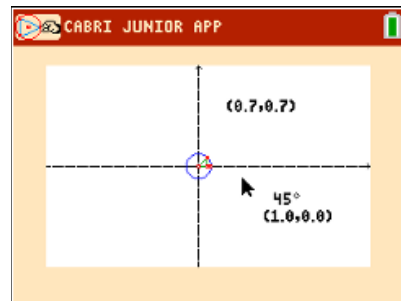
In this activity, students will use the unit circle to examine patterns in the six trigonometric functions. With the aid of the handheld and the file UNITC.8xv, students will compare angles created with the x-axis in all four quadrants and discuss with one another what is happening at each coordinate as they move the point around the circle.


Problem 1 – Searching for Patterns

Using the unit circle, the trig functions can be defined as follows:

$$\sin \theta = \frac{y}{h} \quad \cos \theta = \frac{x}{h} \quad \tan \theta = \frac{y}{x}$$

Using the *Cabri Jr.* application, drag the point on the circle in the first quadrant by pressing the **ALPHA** key and recording the value for $\sin \theta$, $\cos \theta$ and $\tan \theta$ using the displayed x- and y-values, and the equations above.

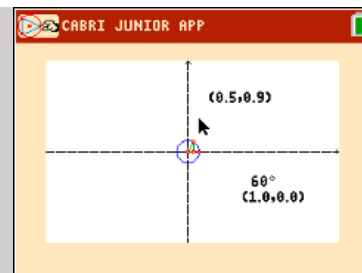


Use the radian conversion to fill in the second column: $\theta \cdot \frac{\pi}{180^\circ}$

Teacher Tip: The angle measures will vary from the exact values in the answer tables due to Cabri Jr. only being able to measure coordinates to one decimal place. Make students aware that they may need to use the closest angle measure possible for some values.

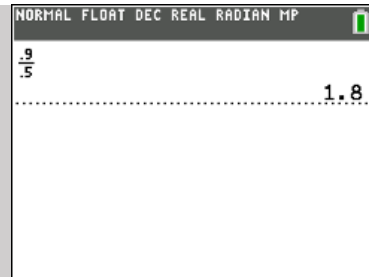
Tech Tip: The Cabri Jr. file “UNITC.8xv”, must be loaded on all calculators before the start of the activity.

Teacher Guidance: Students will move the triangle of the unit circle to find the angle measures listed in the table on the student worksheet. Students will record the values and then answer questions about the patterns in the results. Because *Cabri Jr.* file only measures the angle less than 90° , there is opportunity for some further student learning. This means that when the angle being displayed is 30° and the point is in the second quadrant, the angle being observed is really 150° ($180^\circ - 30^\circ$). This can lead to discussions on reference angles. Students can





write the ratios on their worksheets and then use the Home screen to do their calculations.



Problem 1 – Complete the Table

θ	<i>Radian Measure</i>	$\sin \theta$	$\cos \theta$	$\tan \theta$
0°	0	0	1	0
30°	$\frac{\pi}{6}$	0.5	0.866	0.577
45°	$\frac{\pi}{4}$	0.707	0.707	1
60°	$\frac{\pi}{3}$	0.866	0.5	1.732
90°	$\frac{\pi}{2}$	1	0	Undefined
120°	$\frac{2\pi}{3}$	0.866	-0.5	1.732
135°	$\frac{3\pi}{4}$	0.707	-0.707	-1
150°	$\frac{5\pi}{6}$	0.5	-0.866	-0.577
180°	π	0	-1	0
210°	$\frac{7\pi}{6}$	-0.5	-0.866	0.577
225°	$\frac{5\pi}{4}$	-0.707	-0.707	1
240°	$\frac{4\pi}{3}$	-0.866	-0.5	1.732
270°	$\frac{3\pi}{2}$	-1	0	Undefined
300°	$\frac{5\pi}{3}$	-0.866	0.5	-1.732
315°	$\frac{7\pi}{4}$	-0.707	0.707	-1
330°	$\frac{11\pi}{6}$	-0.5	0.866	-0.577
360°	2π	0	1	0



Teacher Tip: The questions in the document are starting points for discussions about the patterns in the values of trig functions. Teachers should be prepared to discuss patterns beyond the ones in the document.

Problem 2 – Searching for Patterns

Use the values in the table to respond to the following questions.

1. Find the values of θ where $\sin \theta$ is positive.

Solution: $0^\circ < \theta < 180^\circ$ or $0 < \theta < \pi$

2. Find the values of θ where $\cos \theta$ is negative.

Solution: $90^\circ < \theta < 270^\circ$ or $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$

3. Find the values of θ where $\tan \theta$ is positive. Find the values of θ where $\tan \theta$ is negative. Explain.

Solution: Positive: $0^\circ < \theta < 90^\circ$ and $180^\circ < \theta < 270^\circ$ because sin and cos have the same sign
Negative: $90^\circ < \theta < 180^\circ$ and $270^\circ < \theta < 360^\circ$ because sin and cos have different signs

4. Find the angle θ where $\cos \theta = \cos 30^\circ$.

Solution: $\cos 330^\circ$

5. Name two other pairs of angles where the cosine of the angle is the same.

Possible solution: $\cos 45^\circ = \cos 315^\circ$, $\cos 60^\circ = \cos 300^\circ$

6. Find the angle θ where $\tan \theta = \tan 45^\circ$.

Solution: $\tan 225^\circ$

7. Name two other pairs of angles where the tangent of the angle is the same.

Possible solution: $\tan 60^\circ = \tan 240^\circ$, $\tan 30^\circ = \tan 210^\circ$

8. Record all the patterns you see with the sine function.



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Possible solution: Answers will vary. Sample: The values in the first quadrant are repeated in the other quadrants, but have different signs.

9. Describe any other patterns you see.

Possible solution: Answers will vary. Sample: The values of sine and cosine switch within a quadrant, such as $\sin 30^\circ = \cos 60^\circ$, but have different signs.

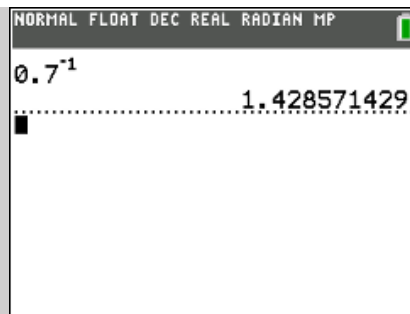
10. Describe what happens at $0^\circ, 90^\circ, 180^\circ, 270^\circ$, and 360° .

Possible discussions: These are the angles that are farthest from the center of the unit circle, and have x and y coordinates of either 0 or 1. These values result in trig ratios that equate to 0, 1, or undefined. Also, 0° and 360° have the same results as they share the same x- and y-coordinates.

11. Explain why the tangent function is undefined for some angle measures.

Possible explanation: Since $\tan \theta = \frac{y}{x}$, anytime $x = 0$ along the unit circle, tangent will be undefined.

Teacher Guidance: In this next problem, students will repeat the activity for the reciprocal trigonometric functions. Students should notice that these functions are reciprocals of the functions from the first part of the activity by looking at the given formulas. This will help them is calculating these functions because they can simply find the reciprocals on the Home screen instead of finding new ratios.



Problem 3 – Patterns in Reciprocal Functions

Using the unit circle, the reciprocal trig functions can be defined as follows:

$$\csc \theta = \frac{h}{y} \quad \sec \theta = \frac{h}{x} \quad \cot \theta = \frac{x}{y}$$

Complete the following table by finding the reciprocals from the computed values on the first table.



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θ	$\csc \theta$	$\sec \theta$	$\cot \theta$
0°	Undefined	1	Undefined
30°	2	1.155	1.732
45°	1.414	1.414	1
60°	1.155	2	0.577
90°	1	Undefined	0
120°	1.155	-2	-1.732
135°	1.414	-1.414	-1
150°	2	-1.155	-1.732
180°	Undefined	-1	Undefined
210°	-2	-1.155	1.732
225°	-1.414	-1.414	1
240°	-1.155	-2	0.577
270°	-1	Undefined	0
300°	-1.155	2	-0.577
315°	-1.414	1.414	-1
330°	-2	1.144	-1.732
360°	Undefined	1	Undefined

Use the values in the table to respond to the following questions.

- Record any patterns that you see.

Possible solution: The same patterns that appeared in the first table are showing up in this table, but now instead of sin forming patterns with cos, csc is forming patterns with sec. Csc is positive and negative in the same quadrants as sin was positive and negative.

- Discuss with a classmate if you notice if any of the functions are undefined. Find which functions and for what values of θ they are undefined.

Possible discussion: Since these three functions are the reciprocals of sin, cos and tan, and since x and y are, at times, zero, this will result in having zero in the denominator and therefore causing each of the three reciprocal trig functions (csc, sec, and cot) to be undefined at either



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$0^\circ, 90^\circ, 180^\circ, 270^\circ$ or 360° .

Further IB Application

In this application, students should use one of the following trig identities and the information used in the previous three problems to answer the questions (a) and (b):

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

- (a) Show that the equation $\cos 2\theta = \cos \theta$ can be written in the form $2\cos^2\theta - \cos \theta - 1 = 0$.

Solution: Replace $\cos 2\theta$ with its identity $2\cos^2\theta - 1$

$$2\cos^2\theta - 1 = \cos \theta$$

Set this equation = 0

$$2\cos^2\theta - \cos \theta - 1 = 0$$

- (b) Hence, solve $\cos 2\theta = \cos \theta$ where $0 \leq \theta \leq \pi$.

Solution: Factor the result from part (a)

$$(2\cos\theta + 1)(\cos\theta - 1) = 0$$

Solve each factor by setting each = 0

$$\begin{aligned} 2\cos\theta + 1 = 0 \quad \text{and} \quad \cos\theta - 1 = 0 \\ \cos\theta = -\frac{1}{2} \quad \text{and} \quad \cos\theta = 1 \\ \theta = 120^\circ \text{ or } \frac{2\pi}{3} \quad \text{and} \quad \theta = 0^\circ \text{ or } 0 \end{aligned}$$

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